UNIVERSITÉ D' ARTOIS

Noncommutative Rings

and

their Applications

LENS 1st-4th JULY 2013

PROGRAM

All talks will take place in room E/8-E/9, building A

Monday

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8h00-9h00: Breakfast (Room E11) and registration (Room E/8-E/9).

9h00-9h10: Welcoming words.

9h15-11h00: Steven Dougherty's course.

11h00-12h00: Alberto Facchini, Padova University (Italy).
Semilocal categories, local functors and applications.

12h00-12h30: Jan Krempa, Warsaw University (Poland).
Every lattice is a lattice of annihilators.

******* 12h30-14h00 Lunch *******

14h00-14h30: Gene Abrams, University of Colorado, Colorado Springs (USA).
Primitive Leavitt path algebras, and a general solution to a question of Kaplansky.

14h30-15h00: Esengul Salturk, Yildiz Technical University Istanbul (Turkey).
Counting Additive Z2 Z4 Codes.

15h00-15h30: Dijana Jakelic, University of North Carolina, Wilmington (USA) and Max Planck Institute for Mathematics, Bonn (Germany).
On q-characters of fundamental representations of quantum affine algebras at roots of unity.

15h30-15h45: Coffee, tea and more!

15h45-16h15: Vladislav Kharchenko, National Autonomous University of Mexico (Mexico).
Quantifications of Kac-Moody algebras.

16h15-16h45: David Jordan, Sheffield University (U.K.).
Simple ambiskew polynomial rings.

16h45-17h15: Marion Candau, Université de Bretagne Occidentale Brest (France).
Non-commutative convolutional codes over the infinite dihedral group.

17h20-17h45: Yuval Ginosar, University of Haifa (Israel)
Maximal isotropic subgroups.
17h50-18h15: M'Hammed Boulagouaz, Faculté des sciences et techniques, Fès (Morroco) and King Khalid University (Saudi Arabia).
About Polynomial Codes.

18h15-18h45: Hamid Usefi, Memorial University St John's, Newfoundland (Canada).
Isomorphism problem for enveloping algebras.

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Tuesday

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8h15-9h00: Breakfast.

9h00-10h55: Steven Dougherty's course.

11h00-12h00: Dolors Herbera, Universitat autonoma de Barcelona (Spain).
Inversion height, rational series and crossed products.

12h00-12h30: Javier Serda, Universidade de São Paulo (Brasilia).
Free group algebras generated by symmetric elements inside division rings with involution.

******* 12h30-14h00 Lunch *******

14h00-14h30: Vladimir Bavula, Sheffied University (U.K.).
Characterizations of left orders in left Artinian rings.

14h30-15h00: Ahmed Cherchem, Université d'Alger, (Algérie).
Recurences over division rings.

15h00-15h30: Daniel Smertnig, University of Graz (Austria).
Sets of lengths in maximal orders in central simple algebras.

15h30-15h35 : Discussions session.
15h35-15h45: Coffee, tea and more!

15h45-16h15: Suat Karadeniz, Fatih University, Istanbul (Turkey).
Extremal binary self-dual codes of length 64 from four-circulant constructions over $\mathbb{F}_2 + u\mathbb{F}_2$.

16h15-16h45: Geoffrey Booth, N. Mandela Metropolitan University (South Africa).
Primeness in near-rings of Continuous Functions.

16h45-17h15: Michal Ziembowski, Warsaw University (Poland).
On classical rings of quotients of duo rings.

17h15-17h30: Discussions sessions.
17h30-17h45: Coffee, tea and more!

17h45-18h45: Bahattin Yildiz, Fatih university Istanbul (Turkey).
Linear codes over $\mathbb{Z}_4 + u\mathbb{Z}_4$]Linear Codes over $\mathbb{Z}_4 + u\mathbb{Z}_4$:Projections, Lifts and formally self-dual codes.

18h15-18h45: Stefan Catoiu, De Paul university, Chicago (USA).
Ideals of the algebra $U(sl_3)$.

18h45-19h00: Conference picture.

19h30- ??: Conference dinner.

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Wednesday

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8h15-9h00: Breakfast.

9h00-10h55: Steven Dougherty's course

11h00-12h00: Patrick Solé, Telecom Paris-Tech (France).
Lower bounds on the minimum distance of long codes in the Lee metric.

12h00-12h30: Mustafa Alkan, Akdeniz University Antalya(Turkey)
Second Modules over Noncommutative Ring.

***** **12h30-14h00 Lunch** *****

14h00-14h30: Mohammed Yousif, Ohio State University, Lima (USA).
Tau-Projective Modules.

14h30-15h00: Stephen Buckley, National University of Ireland Maynooth (Ireland).
Notions of isoclinism for rings, with applications.

15h00-15h30: Pramod Kanwar, University of Ohio, Zanesville (USA)
On idempotents and clean elements in certain ring extensions.

15h30-15h35 : Discussions session.
15h35-15h45: Coffee, tea and more!

15h45-16h15: Christian Lomp, Porto University (Potugal).
Noetherian rings whose injective hulls of simple modules are locally Artinian.

16h15-16h45: Zeynep Odemis-Ozger, Fatih university, Istanbul (Turkey).
Bounds On Codes Over Zps With The Extended Lee Weight.

16h45-17h15: Yiqiang Zhou, Memorial University, Newfoundland (Canada).
On Modules over Group Rings.

17h15-17h30: Discussions sessions.
17h30-17h45: Coffee, tea and more!

17h45-18h45: Djiby Sow, département de mathématiques et informatique, Université Cheik Anta Diop (Senegal).
Gröbner bases over "Dual Euclidean domain".

18h15-18h45: Małgorzata Hryniewicka, Białystok University (Poland).
On rings with finite number of orbits.

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Thursday

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8h15-9h00: Breakfast.

9h00-10h55: Steven Dougherty's course.

11h00-12h00: Edmund Puczyłowski, Warsaw University (Poland).

On prime ideals and radicals of polynomial rings and Z-graded rings.

12h00-12h30: Ofir Schnabel, University of Haifa, Haifa (Israel).

Simply-connected gradings of complex matrix algebras.

***** **12h30-14h00 Lunch** *****

14h00-14h30: Steve Szabo, Eastern Kentucky University (USA).

Algebras Having Bases Consisting Entirely of Units.

14h30-15h00: Cesar Polcino-Miles, Universidade de São Paulo (Brasilia).

Jordan Nilpotency in Group Rings.

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Free afternoon: A visit of the Museum "Louvre Lens" will be organized. One part of the museum is free of charge. The entrance to the exhibition about Rubens is 9 Euros for individual and 8 Euros for groups of at least 10 persons.

Departure from the faculty: 15h20.

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All lectures will take place in room (E8/E9, one double room, Building A).

Breakfasts and coffe, tea,... will be available in room (E 9).

Internet connection will be available both via WiFi and desktops will be available on the third floor of the building.

ABSTRACTS

Primitive Leavitt path algebras, and a general solution to a question of Kaplansky

by

Gene Abrams

University of Colorado Colorado Springs
gabrams@uccs.edu

joint work with Jason Bell and K.M. Rangaswamy

More than forty years ago, Kaplansky posed the question: "Is a regular prime ring necessarily primitive?" A negative answer (via a clever though somewhat ad hoc example) to this question was given by Domanov in 1977.

For any directed graph E and field K, $L_K(E)$ denotes the *Leavitt path K-algebra of E with coefficients in K*. Leavitt path algebras have been defined and subsequently investigated within the past decade. Recall that an algebra A is *left primitive* in case there exists a simple faithful left A-module. In the current work, we give necessary and sufficient conditions on the graph E so that $L_K(E)$ is primitive. In previous work (done by the author and others), necessary and sufficient conditions on E have been established for which $L_K(E)$ is von Neumann regular, as well as for which $L_K(E)$ is prime. Together with the current work, the three results together yield algebras of the type about which Kaplansky queried. We will present four infinite classes of specific examples (both unital and nonunital) of such algebras.

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Second Modules over Noncommutative Ring

by

Mustafa Alkan

Akdeniz University Antalya, Turkey
alkan@akdeniz.edu.tr

Let R be an arbitrary ring. A non-zero unital right R-module M is called a second module if M and all its non-zero homomorphic images have the same annihilator in R. It is proved that if R is a ring such that R/P is a left bounded left Goldie ring for every prime ideal P of R then a right R-module M is a second module if and only if $Q = ann_R(M)$ is a prime ideal of R and M is a divisible right (R/Q)-module. If a ring R satisfies the ascending chain condition on two-sided ideals then every non-zero R-module has a non-zero homomorphic image which is a second module. Every non-zero Artinian module contains second submodules and there are only a finite number of maximal members in the collection of second submodules. Joint Work: S.Ceken and P.F.Smith.

Characterizations of left orders in left Artinian rings.

by

Vladimir Bavula

Sheffield University, U.K.

email:v.bavula@sheffield.ac.uk

Small (1966), Robson (1967), Tachikawa (1971) and Hajarnavis (1972) have given different criteria for a ring to have a left Artinian left quotient ring. In my talk, three more new criteria are given.

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Primeness in near-rings of Continuous Functions.

by

Geoffrey Booth

Nelson Mandela Metropolitan University, Port Elizabeth, South Africa
email:geoffrey.booth@nmmu.ac.za

Let $(G, +)$ be a topological group and let $N(G) := \{f : G \to G \mid f \text{ is continuous}\}$ and let $N_0(G) := \{f : G \to G \mid f \text{ is continuous and } f(0) = 0\}$. Then $N_0(G)$ is a (right) near-ring with respect to the operations of pointwise addition and composition of functions. Note that, if the topology on G is discrete, then $N_0(G) = M_0(G)$, the near-ring of all zero-preserving self-maps of G. A number of different notions of primeness exist in the literature of near-rings, which generalise the concept for non-commutative rings. A notable, if strong definition was given by Booth Groenewald and Veldsman in 1990. A near-ring N is *equiprime* if $a, x, y \in N$, $anx = any$ for all $n \in N$ implies $a = 0$ or $x = y$. This definition is of particular interest because it leads to the only known Kurosh-Amitsur prime radical in the varieties of both zero-symmetric and arbitrary near-rings. In this talk, we will survey recent work done on primeness of tnear-rings of continuous functions and their ideals, and alo study some of the corresponding prime radicals. AMS Subject Classifications: 16Y30, 16N99, 22A99

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Polynomial codes.

by

M'Hammed Boulagouaz

Faculté des sciences et techniques Fès, Morroco and University of King Khalid, Abha, Saudi Arabia
email:boulag@rocketmail.com

In this work we will introduce the notion of polynomial code generalizing the notion of cyclic code. A polynomial code corresponds to a principal ideal in $A[X]/f(X)$. We will construct a generic matrix and a control matrix of a such code. We give a construction of self-dual and BCH code for a given dimension.

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Notions of isoclinism for rings, with applications

by

Stephen Buckley

National University of Ireland Maynooth, Ireland
email:stephen.m.buckley@gmail.com

We discuss a theory of isoclinism for rings and related algebraic structures, and apply this theory to deduce results in combinatorial ring theory. In particular, we investigate the set of values attained by the probability $\Pr(R)$ that a random pair of elements in a finite ring R commute. We characterize all values of $\Pr(R)$ greater than or equal to 11/32; each value strictly greater than 11/32 uniquely identifies the isoclinism class of R, whereas $\Pr(R) = 11/32$ arises from five distinct isoclinism classes. Part of this is joint work with Des MacHale and Áine Ní Shé.

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Non-commutative convolutional codes over the infinite dihedral group.

by

Marion Candau

(Joint work with: Johannes Huisman, Roland Gautier)

Université Européenne de Bretagne et Université de Bretagne Occidentale, France
email:marion.candau@univ-brest.fr

In 1955, Peter Elias introduced convolutional code. The principle of this code is the following. We fix a transfer function τ, which is a binary function with finite support defined on $\mathbb{Z}$. The encoding of a message $f : \mathbb{Z} \to \mathbb{F}_2$ is done by convoluting over $\mathbb{Z}$ with the transfer function :

$$\forall x \in \mathbb{Z},\ c(x) = (f * \tau)(x) = \sum_{y \in \mathbb{Z}} f(y)\tau(x-y)$$

In this talk, we replace the group $\mathbb{Z}$ with the infinite dihedral group D_∞ which is non-commutative and study convolutional codes over this group.

Convolution of two functions being the same operation as the product of two polynomials, we study $\mathbb{F}_2\{X,Y\}/\{X^2 = Y^2 = 1\}$, the polynomial ring of two non-commutative variables with coefficients in $\mathbb{F}_2$ and with $X^2 = Y^2 = 1$. We also adduce the order we have chosen to represent the elements of this ring, which enables to add redundancy to the right and to the left of the message in the systematic form. Moreover, we will see how coding over this group can be represented by two classical convolutions alternating according to the parity of the index of output bits. Furthermore, we adapt Viterbi's algorithm to decode these codes using two different trellis instead of only one, to consider the representation of coding by the two classical alternating convolutions. Finally, we describe properties of these codes, particularly those related to the non-commutativity of the group.

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Ideals of the enveloping algebra of $U(sl_2)$

by

Stefan Catoiu

DePaul University, Chicago, IL 60614 (USA)

email: scatoiu@condor.depaul.edu

Let $\mathfrak{g} = \mathfrak{n}_- \oplus \mathfrak{h} \oplus \mathfrak{n}_+$ be a triangular decomposition of a finite dimensional semisimple Lie algebra $\mathfrak{g}$ in characteristic zero. The adjoint action of $\mathfrak{g}$ extends canonically to an action of the enveloping algebra $U = U(\mathfrak{g})$ on itself. In particular, $U(\mathfrak{n}_+)$ acts adjointly on U. The algebra of invariants $U^{\mathfrak{n}_+}$ can be described by generators and relations as a skew-polynomial algebra over the center Z of U. Since every ideal I of U is generated as an adjoint module by $I \cap U^{\mathfrak{n}_+}$, this leads to finding presentations by generators in $U(\mathfrak{n}_+)$ for the ideals of U. I will show how this process works on $\mathfrak{g} = sl_3$ for prime and primitive ideals, and for annihilators of principal series modules.

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Recurences over division rings.

by

Ahmed Cherchem

USTHB, Alger, Algérie
email:ahmedcherchem@gmail.com

We recall some facts about linear recurring sequences over division rings. Then we prove that the generating function of any sequence over a division ring is rational if and only if the sequence is a linear recurring sequence.

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Course on Coding and rings.

by

Steven Dougherty

Scranton University, Pensylvania (USA)
email:doughertys1@scranton.edu

The foundations of classical coding theory are described with specific attention to the MacWilliams relations and the singleton bound. The transfer of these fundamental ideas to coding theory over over more general alphabets is described. The use of rings in coding theory is also highlighted. We end with a discussion of the major open problems

of coding theory.

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Semilocal categories, local functors and applications.

by

Alberto Facchini

University of Padova, Italy
email:facchini@math.unipd.it

A ring R is *semilocal* if $R/J(R)$ is semisimple artinian, that is, a finite direct product of rings of matrices over division rings. A preadditive category $\mathcal{A}$ is a *null* category if all its objects are zero objects. A preadditive category is *semilocal* if it is non-null and the endomorphism ring of every non-zero object is a semilocal ring. The following are examples of full semilocal subcategories of the category Mod-R of all right modules over an associative ring R:

1. the full subcategory of all artinian right R-modules (Camps and Dicks);
2. the full subcategory of all finitely generated R-modules, for R a semilocal commutative ring (Warfield);
3. the full subcategory of all finitely presented modules right R-modules, for R a semilocal ring (Facchini and Herbera);
4. the full subcategory of all serial modules of finite Goldie dimension (Herbera and Shamsuddin);
5. the full subcategory of all modules of finite Goldie dimension and finite dual Goldie dimension (Herbera and Shamsuddin).

An additive functor $F\colon \mathcal{A} \to \mathcal{B}$ between preadditive categories $\mathcal{A}$ and $\mathcal{B}$ is said to be a *local functor* if, for every morphism $f\colon A \to A'$ in $\mathcal{A}$, $F(f)$ isomorphism in $\mathcal{B}$ implies f isomorphism in $\mathcal{A}$. This notion must not be confused with the notion of *isomorphism reflecting* functor: for every A, A' objects of $\mathcal{A}$, $F(A) \cong F(A')$ implies $A \cong A'$.

We will present the interplay between the concepts of semilocal category, local functor, Jacobson radical of the category and maximal ideals. Our may concern will be the study of the canonical functor $\mathcal{A} \to \mathcal{A}/\mathcal{I}_1 \times \cdots \times \mathcal{A}/\mathcal{I}_n$, where $\mathcal{A}$ is a preadditive category and $\mathcal{I}_1, \ldots, \mathcal{I}_n$ are ideals of $\mathcal{A}$. We will consider and characterize the case where this canonical functor is local. An application will be given. We will conclude with a discussion about Birkhoff's Theorem.

References

[1] A. Alahmadi; A. Facchini, Some remarks on categories of modules modulo morphisms with essential kernel or superfluous image. To appear in *J. Korean Math. Soc.* (2012).

[2] A. Facchini, Direct-sum decompositions of modules with semilocal endomorphism rings. *Bull. Math. Sci.* **2**, 225–279 (2012).

[3] A. Facchini; M. Perone, Maximal ideals in preadditive categories and semilocal categories. *J. Algebra Appl.* **10**(1), 1–27 (2011).

[4] A. Facchini; P. Příhoda, The Krull-Schmidt Theorem in the case two. *Algebr. Represent. Theory* **14**, 545–570 (2011).

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Inversion height, rational series and crossed products.

by

Dolors Herbera

Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), Spain

email:dolors@mat.uab.cat

Joint work with J. Sànchez.

Let $\varepsilon\colon R \hookrightarrow F$ be an embedding of a non-necessarily commutative domain R into a (skew) field F. The subfield of F generated by R is $L = \bigcup_{n\geq 0} R_n$ where $R_0 = R$ and, for $n \geq 1$, R_n is the subring of F generated by the elements of R_{n-1} and the inverses of nonzero elements of R_{n-1}. Notice that we have a chain

$$R_0 \subseteq R_1 \subseteq \cdots \subseteq R_n \subseteq \cdots \qquad (*)$$

By definition, the inversion height of ε is infinite if $(*)$ is not stationary. Otherwise, we say the inversion height of ε is finite.

Let X be a set with at least two elements, and let k be any commutative field. We prove that the inversion height of the embedding $k\langle X\rangle \hookrightarrow D$, where D denotes the universal division ring of fractions of the free algebra $k\langle X\rangle$, is infinite. Therefore, if H denotes the free group on X, the inversion height of the embedding of the group algebra $k[H]$ into the Mal'cev Neumann series ring is also infinite. This answers in the affirmative a question posed by Neumann in 1949.

The case of an infinite set was already settled by Reutenauer in 1996, our result is an application of Reutenauer's one. For a finite set X, there are many embeddings of a free algebra over an infinite set, S say, inside $k\langle X\rangle$. Essentially, we show that if $k\langle X\rangle$ can be given a *Lie Algebra crossed product* structure over S then the universal division rings of fractions of S can be embedded inside the universal division rings of fractions of $k\langle X\rangle$ and the embedding preserves the inversion height. We settle Neumann's question showing that such crossed product structures are relatively frequent. We also show that the same type of argument can be done seeing D as the universal field of fractions of the group algebra over the free group. In this case, a suitable *group crossed product* structure allows us to prove the same kind of result.

Having infinite inversion height is not a distinctive feature of the free field. We show that there is an infinite family of examples of non-isomorphic division ring of fractions of $k\langle X\rangle$ with infinite inversion height.

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On rings with finite number of orbits.

by

Małgorzata Hryniewicka

Institute of Mathematics, University of Białystok, Poland
email: margitt@math.uwb.edu.pl

Based on a joint work with Jan Krempa

Let R be an associative unital ring with the unit group $U(R)$. The additive group of R is denoted by R^+. In [Yasuyuki Hirano, *Rings with finitely many orbits under the regular action,* Lecture Notes in Pure and Appl. Math. 236, Dekker, New York 2004, 343–347], the author concentrated on the left regular group action of $U(R)$ on R^+ defined by $a \rightharpoonup x = ax$ for all $a \in U(R)$, $x \in R$. The main result of this paper asserted the equivalence of the following statements: (i) R has only a finite number of orbits under the left regular action of $U(R)$ on R^+; (ii) R has only a finite number of left ideals. In the talk we will consider the more general group action of $U(R) \times U(R)$ on R^+ defined by

$$(a, b) \rightharpoonup x = axb^{-1}, \tag{0.1}$$

for all $a, b \in U(R)$, $x \in R$. The action (0.1) induces in a natural way an action of the group $U(R) \times U(R)$ on the set of left (respectively, principal left) ideals of R and of ideals of R, however the action on the latter set is trivial. Orbits under the action (0.1) are called simply U-orbits. We introduce the following properties:

FNE R has only a finite number of U-orbits of elements.

FNPLI R has only a finite number of U-orbits of principal left ideals.

FNLI R has only a finite number of U-orbits of left ideals.

FNI R has only a finite number of U-orbits of ideals (R has only a finite number of ideals).

We can directly verify the following connections between the above properties:

$$FNE \Rightarrow FNPLI \Rightarrow FNI \quad \text{and} \quad FNLI \Rightarrow FNPLI \Rightarrow FNI. \tag{0.2}$$

Since for any division ring D and any positive integer n, the $n \times n$ matrix ring $M_n(D)$ has exactly $n+1$ U-orbits both of elements and of left ideals, it follows that *every semisimple artinian ring satisfies all the properties listed in Formula (0.2).* In the talk we will discuss two questions. One of them is: *Under which conditions does a left and/or right artinian ring satisfy FNE or a similar property?* The other is: *Must every ring satisfying FNE or a similar property be left and/or right artinian, or at least semiprimary?*

This talk is an outgrowth of our joint paper with Jan Krempa entitled "On rings with finite number of orbits", which will appear in Publicacions Matemàtiques.

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On q-characters of fundamental representations
of
quantum affine algebras at roots of unity.

by

Dijana Jakelic

Affiliation: University of North Carolina, Wilmington, USA
and
Max Planck Institute for Mathematics, Bonn, Germany
email:jakelic@mpim-bonn.mpg.de

joint work with A. Moura

The notion of q-characters, introduced by E. Frenkel and N. Reshetikhin in the context of finite- dimensional representations of quantum affine algebras, is a generalization of the notion of characters of finite-dimensional representations of simple Lie algebras. The q-characters have been studied extensively in the last 10 years using geometric, combinatorial, and representation-theoretic methods.

In this talk, we will focus on a joint work with A. Moura where we consider the q-characters in the roots of unity setting. We will start by reviewing the basics on finite-dimensional representations of quantum affine algebras at roots of unity including the classification of the irreducible modules in terms of dominant ℓ-weights, the existence of the Weyl modules, and specialization of modules.

It is known that the q-characters are not invariant under the action of the braid group in general. However, we will present a result saying that, if the underlying Lie algebra is of classical type, the q- characters of fundamental representations satisfy a certain invariance property with respect to the braid group action. We will then show how this result leads to explicit formulae for the q-characters of fundamental representations in the root of unity setting.

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Simple ambiskew polynomial rings.

by

David Jordan

Sheffield university, U.K.
email:d.a.jordan@sheffield.ac.uk

Simplicity criteria skew polynomial rings of the form $A[x;\delta]$, where δ is a derivation of an algebra A over a field of characteristic zero, are well-known and there are familiar examples, in particular the Weyl algebras. Although there are results, for example by Lam and Leroy, on simplicity of skew polynomial rings of the more general form $A[x;\alpha,\delta]$, where α is an automorphism and δ is an α-derivation, there are few well-documented examples in which α is not inner. I will report on a longstanding project on simplicity criteria for a wide class of iterated skew polynomial rings which include, as prototypes, the Weyl algebras. This class includes many quantum algebras and some low-dimensional symplectic reflection algebras. As a sample question where simplicity needs a little bit of thought, think of $\mathbb{C}$ as an $\mathbb{R}$-algebra with conjugation as an $\mathbb{R}$-automorphism $z \mapsto \overline{z}$. Take the $\mathbb{R}$-algebra extension of $\mathbb{C}$ generated by x and y subject to the relations $xz = \overline{z}x, yz = \overline{z}y$ (for all $z \in \mathbb{C}$) and $xy - \rho yx = \lambda$ where $0 \neq \rho$ is real and λ is complex and ask when this skewed Weyl algebra is simple.

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On idempotents and clean elements in certain ring extensions.

by

Kanwar Pramod

Ohio University, Zanesville, OH, USA
email:pkanwar@math.ohiou.edu

Joint work with A. Leroy and J. Matczuk

An element a of a ring R is called clean if $a = e + u$ for some idempotent e and some unit u in R. Some results on clean elements and idempotents in group algebras, polynomial rings, power series rings, etc. are given. Among other things, we give conditions under which some elements in the group algebra of infinite dihedral group are clean. For a ring R, it is shown that R is abelian if and only if there exists a positive integer n such that $R[x]$ does not contain any idempotent which is a polynomial of degree n. The conditions under which idempotents of a polynomial ring are conjugate to idempotents in the base ring are also given. It is shown that if R is a 2-primal ring such that $R[x]$ is an ID-ring then every idempotent $e \in M_n(R)[x]$ is conjugate to a diagonal matrix of the form $diag(e_1, \ldots, e_n) \in M_n(R)$, where e_i's denote idempotents in R. Using our results, we give short and new proofs of some classical results.

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Extremal binary self-dual codes of length 64
from four-circulant constructions over $\mathbb{F}_2 + u\mathbb{F}_2$.

by

Suat Karadeniz

Department of Mathematics, Fatih University, 34500, Istanbul, Turkey
skaradeniz@fatih.edu.tr
Joint work with Bahattin Yildiz

A classification of all four-circulant extremal codes of length 32 over $\mathbb{F}_2 + u\mathbb{F}_2$ is done by using four-circulant binary self-dual codes of length 32 of minimum weights 6 and 8. As Gray images of these codes, a substantial number of extremal self-dual codes of length 64 are obtained. In particular a new code with $\beta = 80$ in $W_{64,2}$ is found. All the extremal codes that are found by this method are tabulated.

keywords: Lee weight, Gray maps, extremal self-dual codes, projections, lifts, four-circulant codes

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Quantifications of Kac-Moody algebras.

by

Vladislav Kharchenko

Universidad Nacional Autónoma de México, FES- Cuautitlán, MEXICO
email: vlad@unam.mx

We analyze in what extent a quantum universal enveloping algebra of a Kac-Moody algebra $\mathfrak{g}$ is defined by multi-degrees of its defining relations. To this end we consider a class of character Hopf algebras defined by the same number of defining relations of the same degrees as the Kac-Moody algebra $\mathfrak{g}$ is. Certainly, this class contains all possible quantizations of $\mathfrak{g}$ (including multiparameter ones) and there is a reason to consider these Hopf algebras as quantum deformations of the universal enveloping algebra of $\mathfrak{g}$ as well. There is a number of books and a great number of articles concerning quantifications of Lie algebras. However almost each publication has its own modifications in construction and different notations so that sometimes it is not quite clear if results of one work may be applied to a construction of another. Since in all the constructions related to $\mathfrak{g}$ the number and the degrees of relations are the same, the unification in the above class provides a possibility to understand the differences, if any, of these constructions just comparing the basic invariants inside that class.

We show that if the generalized Cartan matrix A of $\mathfrak{g}$ is connected then the algebraic structure of a quantification, up to a finite number of exceptional cases, is defined by just one “continuous” parameter q related to a symmetrization of A, and one “discreet” parameter $\mathfrak{m}$ related to modular symmetrizations of A. The Hopf algebra structure is defined by additional $n(n-1)/2$ “continuous” parameters. We also consider more fully the exceptional cases for Cartan matrices of finite or affine type counting the number of exceptional values of parameters in terms of the Fibonacci sequence.

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Every lattice is a lattice of annihilators.

by

Jan Krempa

University of Warsaw, Poland
email:jkrempa@mimuw.edu.pl

In this talk $\mathbb{K}$ will be a field and A an associative $\mathbb{K}$-algebra with $1 \neq 0$. It is known that the set $\mathcal{I}_l(A)$ of all left ideals and the set $\mathcal{I}_r(A)$ of all right ideals are complete modular lattices under inclusion.

Let $\mathcal{A}_l(A)$ be the set of all left annihilators in A and let $\mathcal{A}_r(A)$ be the set of all right annihilators in A. Then $\mathcal{A}_l(A) \subseteq \mathcal{I}_l(A)$ and $\mathcal{A}_r(A) \subseteq \mathcal{I}_r(A)$ are complete lattices under inclusion, but they need not be sublattices.

In several papers QF-algebras and other algebras with $\mathcal{I}_l(A) = \mathcal{A}_l(A)$ were investigated. On the other hand, some strange properties of the lattice $\mathcal{A}_l(A)$ were exhibited. In this talk I'm going to present the following result:

Theorem 0.1 (with M. Jastrzębska)**.** *Let L be a lattice. then there exists an algebra A_L and a natural lattice embedding of L into $\mathcal{A}_l(A_L)$. If L is finite then we can assume that A_L is finite dimensional.*

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Noetherian rings whose injective hulls of simple modules are locally Artinian.

by

Chrstian Lomp

Porto University, Portugal
email:clomp@fc.up.pt

The Jacobson's conjecture is an open problem in ring theory and asks whether the intersection of the powers of the Jacobson radical of a two-sided Noetherian ring is zero. Jategaonkar answered the conjecture in the affirmative for a Noetherian ring R under an additional assumption (called FBN) which in particular implies that any finitely generated essential extension of a simple left R-module is Artinian. The latter condition, denoted by $(\diamond)$, is a sufficient condition for a positive answer to the Jacobson's conjecture. In this talk I will discuss the question which Noetherian algebras do or do not satisfy condition $(\diamond)$. I will quickly review the history of property $(\diamond)$. Then I will explicitly characterize those enveloping algebras of finite dimensional complex nilpotent Lie superalgebras that do satisfy $(\diamond)$ and will consider differential operator rings $R[x; d]$ with R a commutative Noetherian ring and d is a derivation.

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Bounds On Codes Over $\mathbb{Z}_{p^s}$ With The Extended Lee Weight.

by

Zeynep Odemis-Ozger

Fatih university, Istanbul (Turkey)
email:zynp.odemis@gmail.com

joint work with Bahattin Yildiz.

We introduce a new generalization of the Lee weight to Zps and GR(ps;m) with its Gray map, which is formulated as follows:

$$w_L(x) := \begin{cases} x, & \text{if } x \leq p^{s-1}, \\ p^{s-1}, & \text{if } p^{s-1} \leq x \leq p^s - p^{s-1}, \\ p^s - x, & \text{if } p^s - p^{s-1} < x \leq p^s - 1, \end{cases}$$

and lift this weight function to $GR(p^s, m)$ as well as its Gray map, and investigate bounds for the dimension of the kernel and the rank of the image of a code over $\mathbb{Z}_{p^s}$.

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Jordan Nilpotency in Group Rings.

by

C. Polcino Milies

Universidade Federal do ABC and the Universidade de São Paulo, Brazil.
email:polcinomilies@gmail.com

Joint work with E.G. Goodaire.

The *Lie bracket* of an associative algebra A is the ring commutator $[x, y] = xy - yx$. Using this bracket to define a new product in A, it becomes a Lie Algebra. This algebra is *nilpotent* of index $n \geq 2$ if this is the smallest positive integer such that $[\ldots[[x_1, x_2], x_3], \ldots, x_n] = 0$ for all $x_1, x_2, \ldots, x_n \in A$.

Let $A = RG$ denote the group ring of a group G over a commutative ring R, with unity. Further, assume that $\alpha \mapsto \alpha^*$ is an involution on RG which is a linear extension of an involution in G. Then, the set $A^- = \{a \in A \mid a^* = -a\}$ of *skew-symmetric* elements of A is a Lie subring.

Similarly, if we define in A a product by $x \circ y = xy + yx$, then it becomes a Jordan ring and the set $A^+ = \{a \in A \mid a^* = a\}$ of *symmetric* elements is a Jordan subring of A. A is called *Jordan nilpotent* of index $n \geq 2$ if n is the smallest positive integer such that $(\ldots((x_1 \circ x_2) \circ x_3) \cdots) \circ x_n = 0$ for all $x_1, x_2, \ldots, x_n \in A$.

Questions such as when is A Lie nilpotent or when is A^- Lie nilpotent have been discussed in recent literature [1, 2, 5, 6].

We shall discuss similar questions for Jordan nilpotency.

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On prime ideals and radicals of polynomial rings and Z-graded rings..

by

Edmund Puczylowski

University of Warsaw, Poland
email:edmundp@mimuw.edu.pl

The topics indicated in the title are studied very extensively since a long time. However there are still many open problems in the area. The aim of the talk is to present some new results and questions concerning the subject. We will concentrate on results and problems related to classical radicals (Jacobson, nil, Brown-McCoy).

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Counting Additive Z2 Z4 Codes..

by

Esengul Salturk

Yildiz Technical University Istanbul, Turkey
email: mathematicianesen@gmail.com

joint work with Steven T. Dougherty

It is well known, from the work of Delsarte, that the abelian groups in the binary Hamming scheme correspond to additive $\mathbb{Z}_2\mathbb{Z}_4$ codes. These codes give rise to propelinear binary codes via a Gray map. Specifically, there a distance preserving map from $\mathbb{Z}_2^\alpha\mathbb{Z}_4^\beta$ to the binary space such that the image behaves like a linear code without necessarily being linear. This relationship has been used to obtain many interesting results. In this paper, we define free codes in this space and count the number of free codes. We then count the number of arbitrary $\mathbb{Z}_2\mathbb{Z}_4$ linear codes of type $(\alpha, \beta; \gamma, \delta : \kappa)$

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Simply-connected gradings of complex matrix algebras

by

Ofir Schnabel

Haifa University, Israel
email:os2519@yahoo.com

A joint work with Y. Ginosar.

The simply-connected Galois coverings of a linear K-category $\mathtt{B}$ are in one-to-one correspondence with the connected gradings of $\mathtt{B}$ which are maximal with respect to quotient gradings. By a structure theorem of Bahturin, Seghal and Zaicev, there is one-to-one correspondence between simply-connected Galois coverings of the one-object $\mathbb{C}$-category $M_n(\mathbb{C})$ (up to equivalence of Galois coverings), and the set of pairs $X_n = \{(G, \gamma)\}$, where G is a group of order dividing n^2 and γ is an $\mathrm{Aut}(G)$-orbit of a nondegenerate cohomology class in $H^2(G, \mathbb{C}^*)$. For $n \geq 2$, clearly $|X_n| \geq 2$, in particular there is no universal covering of $M_n(\mathbb{C})$ as observed by Cibils, Redondo and Solotar. Moreover, they compute the inverse limit of the diagram of connected gradings of $M_p(\mathbb{C})$ (where p is prime) with respect to quotient gradings, namely the intrinsic fundamental group of $M_p(\mathbb{C})$. In these cases $|X_p| = 2$.
In this talk we extend some of these results and discuss an approach for computing the fundamental group of $M_n(\mathbb{C})$ for more general n.

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Free group algebras generated by symmetric elements
inside
division rings with involution.

by

Javier Serda

Universidade de São Paulo, Brazil
email:jsanchezserda@gmail.com

A joint work with Vitor O. Ferreira and Jairo Z. Gonçalves..

We use a result of I. Hughes to extend involutions of crossed products group rings to certain division rings generated by them (that need not be Ore domains). As a consequence, we obtain that if k is a field, $(G, <)$ an ordered group and $k[G]$ the group ring, then the *canonical involution*

$$k[G] \to k[G] \qquad \sum_{x\in G} x a_x \mapsto \sum_{x\in G} x^{-1} a_x,$$

can be extended to $k(G)$, the division ring generated by $k[G]$ inside its Malcev-Neumann series ring $k((G, <))$.

We then use a result of J. Bell and D. Rogalski to show that the classical Ore division ring of fractions of a group ring $k[G]$, with G a torsion-free nilpotent group of index two, contains a noncommutative free group algebra generated by symmetric elements with respect to the canonical involution.

Using these results we prove the following theorem.

Theorem: Let k be a field, $(G, <)$ an ordered group, $k[G]$ the group ring, and $k(G)$ the division ring generated by $k[G]$ inside $k((G, <))$. Then the following are equivalent:

(1) $k(G)$ contains a noncommutative free group algebra generated by symmetric elements with respect to the canonical involution.

(2) G is not abelian.

(3) $k(G)$ is not locally P. I.

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Sets of lengths in maximal orders in central simple algebras.

by

Daniel Smertnig

Institut für Mathematik und wissenschaftliches RechnenKarl-Franzens-Universität Graz

email: daniel.smertnig@uni-graz.at

In a noetherian domain R every element can be expressed as a finite product of irreducibles. Such a factorization is, in general, highly non-unique. However, if R is commutative, then the set of lengths for each $a \in R \setminus \{ 0 \}$ is finite and non-empty. In the commutative setting there is a long tradition of studying the non-uniqueness of factorizations by means of various arithmetical invariants.

Let A be a central simple algebra over a global field K, and let R be a maximal order in A. For a non-unit non-zerodivisor $a \in R$, its set of lengths, denoted $\mathsf{L}(a)$, consists of all $k \in \mathbb{N}$ such that there exist irreducibles $u_1, \ldots, u_k \in R$ with $a = u_1 \cdot \ldots \cdot u_k$. The system of sets of lengths of R is then defined as

$$\mathcal{L}(R) = \left\{ \mathsf{L}(a) \mid a \in R \text{ a non-unit non-zerodivisor} \right\}.$$

Building on ideas from the commutative setting of Krull domains, we use the one-sided divisorial ideal theory of R and its Asano-equivalence class of maximal orders to investigate $\mathcal{L}(R)$. These ideals form a Brandt groupoid, and we observe a connection between the factorizations of a non-zerodivisor $a \in R$ and the factorizations of the principal ideal Ra into maximal integral (one-sided) ideals in this groupoid.

In the case that every stably free left R-ideal is free, we can construct a transfer homomorphism to a monoid of zero-sum sequences over a ray class group. This is similar to the case of a commutative Krull domain with finite class group, and immediately gives many finiteness and structural results on the sets of lengths in R. On the other hand, if this condition is violated and K is a number field, then A is necessarily a totally definite quaternion algebra and an ideal-theoretic construction shows that sets of lengths exhibit distinctly different features.

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Lower bounds on the minimum distance of long codes in the Lee metric.

by

Patrick Solé

Telecom Paris-Tech, France
email:patrick.sole@telecom-paristech.fr

The Gilbert type bound for codes in the title is reviewed, both for small and large alphabets. Constructive lower bounds better than these existential bounds are derived from geometric codes, either over Fp or Fp2 ; or over even degree extensions of Fp: In the latter case the approach is concatena- tion with a good code for the Hamming metric as outer code and a short code for the Lee metric as an inner code. In the former case lower bounds on the minimum Lee distance are derived by algebraic geometric arguments inspired by results of Wu, Kuijper, Udaya (2007).

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Gröbner bases over "Dual Euclidean domain".

by

Djiby Sow

Université Cheikh Anta Diop, Dakar SENEGAL
email:sowdjibab@yahoo.fr

Several methods for computing Gröbner bases over rings with zero divisors were already proposed, but such rings don't include the ring $A[\varepsilon]$ satisfying to $\varepsilon^2 = 0$ where A is an Euclidean domain. In this paper we have successfully designed Buchberger's algorithm over $A[\varepsilon][X_1, ...X_m]$. We give some applications in $\mathbb{Z}[\varepsilon]$ and $K[x][\varepsilon]$ where K is a field.

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Algebras Having Bases Consisting Entirely of Units.

by

Steve Szabo

Eastern Kentucky University (USA)
email:steve.szabo@eku.edu

A joint work with Sergio Lopez, Jeremy Moore and Nick Pilewski.

We introduce a hierarchy of notions about algebras having a basis $\mathcal{B}$ consisting entirely of units. Such a basis is called an invertible basis and algebras that have

invertible bases are said to be invertible algebras. The other conditions considered in the said hierarchy include the requirement that for an invertible basis $\mathcal{B}$, the set of inverses $\mathcal{B}^{-1}$ be itself a basis, the notion that $\mathcal{B}$ be closed under inverses and the idea that $\mathcal{B}$ be closed under products. Included in the hierarchy are extension fields, full matrix rings and crossed products.

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Isomorphism problem for enveloping algebras.

by

Hamid Usefi

Memorial University St John's, Newfoundland, Canada
email:usefi@mun.ca

Let L be a Lie algebra with universal enveloping algebra $U(L)$. A particular invariant of L is said to be determined by $U(L)$ if every Lie algebra H also possesses this invariant whenever $U(L)$ and $U(H)$ are isomorphic as associative algebras. Thus, roughly speaking, an invariant of L is determined by $U(L)$ whenever it can be deduced from the algebraic structure of $U(L)$ without any direct knowledge of the underlying Lie algebra L itself. For example, it is well-known that the dimension of a finite-dimensional Lie algebra L is determined by $U(L)$ since it coincides with the Gelfand-Kirillov dimension of $U(L)$. In this talk we demonstrate that certain other invariants of L are also determined by $U(L)$. We also discuss the most far reaching problem of this sort that asks whether or not (the isomorphism type of) every Lie algebra L is determined by $U(L)$.

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Linear Codes over $\mathbb{Z}_4 + u\mathbb{Z}_4$:Projections, Lifts and formally self-dual codes

by

Bahattin Yildiz

Fatih University, Istanbul, Turkey
email: byildiz@fatih.edu.tr

A joint work with Karadeniz Suat

We consider linear codes over $\mathbb{Z}_4 + u\mathbb{Z}_4$, a non-chain extension of $\mathbb{Z}_4$. We define Lee weights, Gray maps for these codes and we prove the MacWilliams identities for the complete, symmetrized and Lee weight enumerators. We consider projections to the rings $\mathbb{Z}_4$ and $\mathbb{F}_2 + u\mathbb{F}_2$ and we study self-dual codes over $\mathbb{R}$ in connection with these projections. Finally we give two constructions for formally self-dual codes over $\mathbb{R}$ and their $\mathbb{Z}_4$-images together with some examples of codes.

Keywords: complete weight enumerator, MacWilliams identities, projections, lifts, formally self-dual codes, codes over rings

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Tau-Projective Modules.

by

Mohamed Yousif

The Ohio State University at Lima
email:

A joint work with Ismail Amin and Yasser Ibrahim of Cairo University..

According to T. Nakayama a ring R is called quasi-Frobenius (QF-ring) if R is left (or right) artinian and if $\{e_1, e_2, \cdots, e_n\}$ is a basic set of primitive idempotents of R, then there exists a (Nakayama) permutation σ of $\{1, 2, \cdots, n\}$ such that $soc(Re_k) \cong Re_{\sigma k}/Je_{\sigma k}$ and $soc(e_{\sigma k}R) \cong e_kR/e_kJ$, where $J = J(R)$ is the Jacobson radical of R. This remarkable description by Nakayama reduces the perfect duality in QF-rings to a duality between the Jacobson radical and the socle of the indecomposable projective components of the basic subring of R. Nakayama's result was the primary motivation behind the introduction of the concepts of *soc-injective* modules and its dual *rad-projective* modules.

In this talk we introduce the notion of τ-projective modules relative to any pre-radical τ, where a right R-module M is called τ-N-projective if $\sigma : N \to K$ is an R-epimorphism with $\tau(N) \hookrightarrow \ker\sigma$, then every homomorphism $f : M \to K$ can be lifted to a homomorphism $\lambda : M \longrightarrow N$ such that $g\lambda = f$. The module M is called τ-projective (resp., τ-quasi-projective) if M is τ-R_R-projective (resp., τ-M-projective), and is called strongly τ-projective if it is τ-N-projective for every right R-module N.

New and interesting results are obtained when $\tau(M) = rad(M)$, $soc(M)$ or $\delta(M)$, where $rad(M)$, $soc(M)$ and $\delta(M)$ denotes to the radical, the socle and the δ-submodule of M, respectively. We start our talk by highlighting all the interesting properties of these new notions and show with examples that these notions are non trivial and natural extensions of projective modules. For example, the class of (strongly) τ-projective right R-modules is closed under isomorphisms, direct sums and summands. The notion of τ-projective cover is introduced and new characterizations of semiperfect and perfect rings in terms of τ-projective covers are provided.

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On Modules over Group Rings.

by

Yiqiang Zhou

Memorial University of Newfoundland, St.John's, Canada
email: zhou@mun.ca

A joint work with M.Tamer Kosan, Tsiu-Kwen Lee.

Let M be a right module over a ring R and let G be a group. The set MG of all formal finite sums of the form $\sum_{g\in G} m_g g$ where $m_g \in M$ becomes a right module over the group ring RG under addition and scalar multiplication similar to the addition and multiplication of a group ring. In this paper, we study basic properties of the RG-module MG, and characterize module properties of $(MG)_{RG}$ in terms of properties of M_R and G. Particularly, we prove the module-theoretic versions of several well-known results on group rings, including the Maschke's Theorem and the classical characterizations of right self-injective group rings and of von Neumann regular group rings.

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On classical rings of quotients of duo rings

by

Michał Ziembowski

Warsaw University of Technology, Poland
m.ziembowski@mini.pw.edu.pl

In this talk we will construct a duo ring R such that the classical right ring of quotients $Q^r_{cl}(R)$ of R is neither right nor left duo. This gives a negative answer to [1, Question 1].

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ADDENDUM

Maximal isotropic subgroups

by

Yuval Ginosar

Haifa University, Israel
ginosar@math.haifa.ac.il

Based on joint works with N. Ben David and E. Meir

The analog of Lagrangians for symplectic forms over finite groups is motivated by the fact that symplectic G-forms with a normal Lagrangian $N \triangleleft G$ are in one-to-one correspondence, up to inflation, with bijective 1-cocycle data on the quotients G/N.

This yields a method to construct groups of central type from such quotients, known as Involutive Yang-Baxter groups.